$$\int x^2\, e^x dx = \begin{Bmatrix} u = x^2 & \rightarrow & du = 2x dx \\ dv = e^x dx & \rightarrow & v = e^x \end{Bmatrix} = x^2\, e^x - \int 2x\, e^x dx = (*)$$

$$\int 2x\, e^x dx = \begin{Bmatrix} u = 2x & \rightarrow & du = 2 dx \\ dv = e^x dx & \rightarrow & v = e^x \end{Bmatrix} = 2x\, e^x - \int 2\, e^x dx = 2x\, e^x - 2 \int e^x dx =$$

$$= 2x\, e^x - 2\, e^x$$

$$(*) = x^2\, e^x - \left(2x\, e^x - 2\, e^x\right) + C = x^2\, e^x - 2x\, e^x + 2\, e^x + C$$

$Solución:\quad \int x^2\, e^x dx = x^2\, e^x - 2x\, e^x + 2\, e^x + C$

$$\int e^x\, sen\, x\, dx = \begin{Bmatrix} u = sen\, x & \rightarrow & du = \cos x\, dx \\ dv = e^x\, dx & \rightarrow & v = e^x \end{Bmatrix} = e^x\, sen\, x - \int e^x \cos x\, dx = (*)$$

$$\int e^x \cos x\, dx = \begin{Bmatrix} u = \cos x & \rightarrow & du = -sen\, x\, dx \\ dv = e^x\, dx & \rightarrow & v = e^x \end{Bmatrix} = e^x \cos x - \int -e^x\, sen\, x\, dx =$$

$$= e^x \cos x + \int e^x\, sen\, x\, dx$$

$$(*) = e^x\, sen\, x - e^x \cos x - \int e^x\, sen\, x\, dx$$

Hemos obtenido:

$$\int e^x\, sen\, x\, dx = e^x\, sen\, x - e^x \cos x - \int e^x\, sen\, x\, dx$$

$$\int e^x\, sen\, x\, dx + \int e^x\, sen\, x\, dx = e^x\, sen\, x - e^x \cos x$$

$$2\int e^x\, sen\, x\, dx = e^x\, sen\, x - e^x \cos x \quad \rightarrow$$

$Solución:\quad \int e^x\, sen\, x\, dx = \dfrac{e^x\, sen\, x - e^x \cos x}{2} + C$

INTEGRACIÓN DE FUNCIONES RACIONALES.

Ejemplos:

$$\int \frac{dx}{x^2-3x+2}=$$

$$\begin{array}{c|ccc} & 1 & -3 & 2 \\ 1 & & 1 & -2 \\ \hline & 1 & -2 & 0 \end{array} \quad \rightarrow \quad x^2-3x+2=(x-1)(x-2)$$

$$\frac{1}{x^2-3x+2}=\frac{A}{x-1}+\frac{B}{x-2}=\frac{A(x-2)+B(x-1)}{(x-1)(x-2)}=\frac{A(x-2)+B(x-1)}{x^2-3x+2}$$

$$1=A(x-2)+B(x-1) \quad \begin{cases} x=1, & 1=-A \\ x=2, & 1=B \end{cases} \quad \rightarrow \quad A=-1 \quad y \quad B=1$$

$$\frac{1}{x^2-3x+2}=\frac{-1}{x-1}+\frac{1}{x-2} \quad \rightarrow \quad \int \frac{dx}{x^2-3x+2}=\int\left(\frac{-1}{x-1}+\frac{1}{x-2}\right)dx=$$

$$=\int \frac{-1}{x-1}dx+\int \frac{1}{x-2}dx=-Ln|x-1|+Ln|x-2|+C=Ln\left|\frac{x-2}{x-1}\right|+C$$

$$Solución: \quad \int \frac{dx}{x^2-3x+2}=Ln\left|\frac{x-2}{x-1}\right|+C$$

$$\int \frac{1}{(x-2)^2\,(x+3)}dx =$$

$$\frac{1}{(x-2)^2\,(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3} = \frac{A(x-2)(x+3) + B(x+3) + C(x-2)^2}{(x-2)^2\,(x+3)} =$$

$$1 = A(x-2)(x+3) + B(x+3) + C(x-2)^2$$

$$x = 2, \quad 1 = 5B \quad \rightarrow \quad B = \frac{1}{5}$$

$$x = -3, \quad 1 = 25C \quad \rightarrow \quad C = \frac{1}{25}$$

$$x = 0, \quad 1 = -6A + 3B + 4C \quad \rightarrow \quad 1 = -6A + \frac{3}{5} + \frac{4}{25}; \quad \frac{6}{25} = -6A \quad \rightarrow \quad A = \frac{-1}{25}$$

$$\int \frac{1}{(x-2)^2\,(x+3)}dx = \int \frac{\frac{-1}{25}}{x-2}dx + \int \frac{\frac{1}{5}}{(x-2)^2}dx + \int \frac{\frac{1}{25}}{x+3}dx =$$

$$= \frac{-1}{25}\int \frac{1}{x-2}dx + \frac{1}{5}\int \frac{1}{(x-2)^2}dx + \frac{1}{25}\int \frac{1}{x+3}dx = (**)$$

La 1ª y 3ª integral son inmediatas, resolvamos la 2ª,

$$\int \frac{1}{(x-2)^2}dx = \int (x-2)^{-2}\,dx = \frac{(x-2)^{-2+1}}{-2+1} = \frac{(x-2)^{-1}}{-1} = \frac{-1}{x-2}$$

Y finalmente,

$$(**) = \frac{-1}{25}Ln|x-2| + \frac{1}{5}\frac{-1}{x-2} + \frac{1}{25}Ln|x+3| + C = \frac{-1}{25}Ln|x-2| - \frac{1}{5\,(x-2)} + \frac{1}{25}Ln|x+3| + C$$