EJERCICIO A

PROBLEMA 1. Dada la matriz A = $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$, calcula A . $A^t - 5\ A^{-1}$, siendo A^t y A^{-1} las matrices traspuesta e inversa de A, respectivamente.

Solución:

$$A^t = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

Cálculo de A^{-1}

$$|A| = \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = 3 + 2 = 5 \neq 0 \quad \rightarrow \quad \exists\, A^{-1}$$

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \xrightarrow{\alpha_{i\,j}} \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} \xrightarrow{A_{i\,j}} \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix} \xrightarrow{A_{j\,i}} \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$$

$$luego \quad A^{-1} = \frac{1}{5}\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

Entonces,

$$A\,A^t - 5\,A^{-1} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} - 5\begin{pmatrix} \frac{3}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 1.1 + 2.2 & 1(-1) + 2.3 \\ -1.1 + 3.2 & -1(-1) + 3.3 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 5 & 5 \\ 5 & 10 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 7 \\ 4 & 9 \end{pmatrix}$$