EJERCICIO A

PROBLEMA 1. Determina la matriz A que verifica la ecuación $A B + A = 2 B^t$, donde

$B = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}$ y B^t representa la matriz traspuesta de B.

Solución:

$A B + A = 2 B^t$

$A (B + I) = 2 B^t$

Si existe $(B + I)^{-1}$ *entonces* $A = 2 B^t (B + I)^{-1}$

$$B + I = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 0 & 3 \end{pmatrix}$$

$$\begin{vmatrix} 4 & -1 \\ 0 & 3 \end{vmatrix} = 12 \neq 0 \quad luego \ \exists \, (B + I)^{-1}$$

Calculemos la matriz inversa de (B + I)

$$B + I = \begin{pmatrix} 4 & -1 \\ 0 & 3 \end{pmatrix} \xrightarrow{\alpha_{ij}} \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix} \xrightarrow{A_{ij}} \begin{pmatrix} 3 & 0 \\ 1 & 4 \end{pmatrix} \xrightarrow{A_{ji}} \begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix} \rightarrow (B + I)^{-1} = \frac{1}{12}\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{12} \\ 0 & \frac{1}{3} \end{pmatrix}$$

$$Como \quad B = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix} \quad luego, \quad B^t = \begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}$$

Finalmente, calculemos la matriz A, $A = 2 B^t (B + I)^{-1}$

$$A = 2\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} \frac{1}{4} & \frac{1}{12} \\ 0 & \frac{1}{3} \end{pmatrix} = 2\begin{pmatrix} 3\,\frac{1}{4} - 0\,.\,0 & 3\,\frac{1}{12} - 0\,.\,\frac{1}{3} \\ \frac{-1}{4} + 2\,.\,0 & \frac{-1}{12} + 2\,\frac{1}{3} \end{pmatrix} = 2\begin{pmatrix} \frac{3}{4} & \frac{3}{12} \\ \frac{-1}{4} & \frac{-1}{12} + \frac{2}{3} \end{pmatrix} = 2\begin{pmatrix} \frac{3}{4} & \frac{3}{12} \\ \frac{-1}{4} & \frac{-1}{12} + \frac{8}{12} \end{pmatrix} =$$

$$= 2\begin{pmatrix} \frac{3}{4} & \frac{3}{12} \\ \frac{-1}{4} & \frac{7}{12} \end{pmatrix} = \begin{pmatrix} \frac{6}{4} & \frac{6}{12} \\ \frac{-2}{4} & \frac{14}{12} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{7}{6} \end{pmatrix}$$

$$Luego, \ A = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{7}{6} \end{pmatrix}$$